TRINITY COLLEGE

Year 12 Mathematics Applications
Test 3 2021
Graphs and Networks

Section 1 – Calculator Free

STUDENT'S NAME Solutions - Martin

DATE: Thursday 13th May **TIME:** 25 minutes **MARKS:** 27

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

A group of 5 Students wish to organise a 'paper-scissors-rock' tournament. Each student must play against every other student (in the group of 5).

(a) How many matches will need to be played in total? [2]

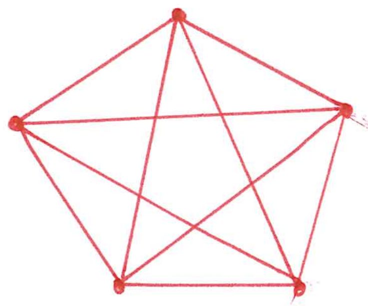
$$\frac{5(5-1)}{2} \checkmark$$

$$= 10 \checkmark$$

(b) What is the name of the type of graph will need to be used to show how the matches are organised? [1]

Complete

(c) Draw a graph to show how the matches are to be played against each other. [2]

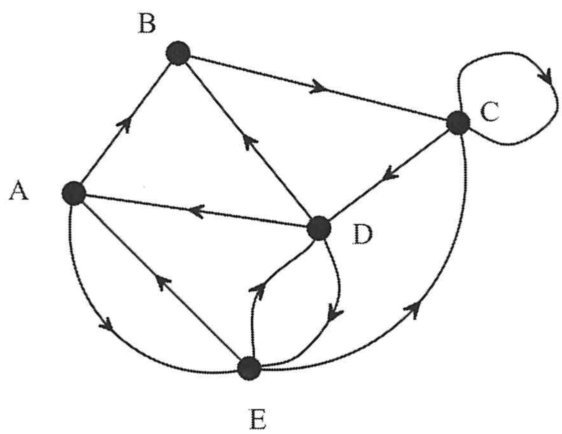


V ✓

E ✓

2. (8 marks)

The digraph below shows the walkways between different tourist attractions.



(a)

(ii) Complete the adjacency matrix 'M' below for the graph. [3]

To

$$\text{From} \quad M = \begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 1\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 1 & 0\\ 1 & 1 & 0 & 0 & 1\\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \quad ✓$$

(i) What does the sum of the values in row 1 represent? [1]

Out degree of vertex A

(b) A visitor walks the following route: ABCDECD. Is this route a Hamiltonian path? Justify your answer. [2]

No ✓

Vertex C is repeated ✓

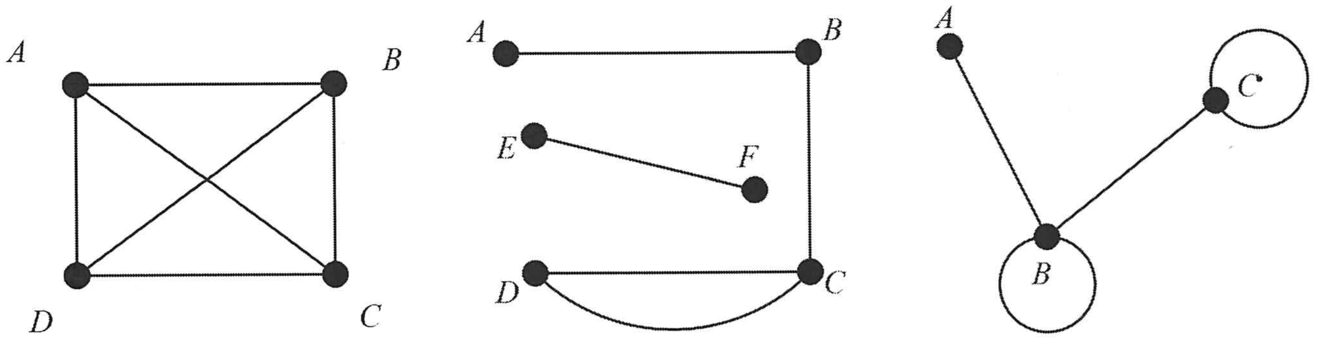
(c) The matrix M^3 has been constructed below. Explain what the entry in row 4, column 3 tells us. [2]

$$M^3 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 2 & 2 \\ 2 & 2 & 4 & 3 & 2 \\ 3 & 2 & 5 & 3 & 2 \end{bmatrix} \end{array}$$

✓
There are 4 'three-step' routes from D to C
✓

3. (9 marks)

Consider the following graphs



Graph 1 Graph 2 Graph 3

(a) Which of the above graphs are NOT simple? Justify. [3]

2 - Multiple edges

3 - loops

2 ✓
3 ✓
Justifications ✓

(b) What is the degree sum of Graph 2? [1]

10

(c) State a walk of length 4 for Graph 1 [1]

A, B, C, D, B (or other options)

(d) Which of the above graphs are connected? [2]

1 and 3

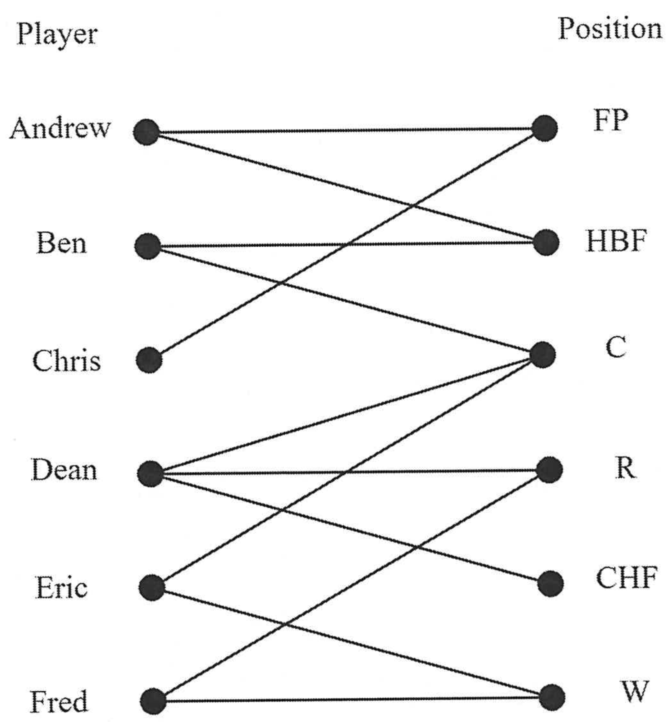
(e) Verify that Graph 3 satisfies Euler's formula. [2]

$3 + 3 - 4 = 2$

$2 = 2$

4. (5 marks)

Six football players: Andrew, Ben, Chris, Dean, Eric and Fred are trying to decide who should play in which of the following positions: Forward Pocket (FP), Half-Back-Flank (HBF), Wing (W), Centre (C), Rover (R), and Centre-Half-Forward (CHF). The following graph shows which positions each player could possibly fill:



(a) What type of graph has been used above to display the position options? [1]

Bipartite

(b) Which player must play 'Half-Back-Flank'? [1]

Andrew

(c) Which position much each of the following players play in?

(i) Dean [1]

CHF

(ii) Fred [1]

R

(iii) Ben [1]

C

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Year 12 Mathematics Applications
Test 3 2021

Section 2 Calculator Assumed

STUDENT'S NAME Solutions - Martin

DATE: Thursday 13th May **TIME:** 25 minutes **MARKS:** 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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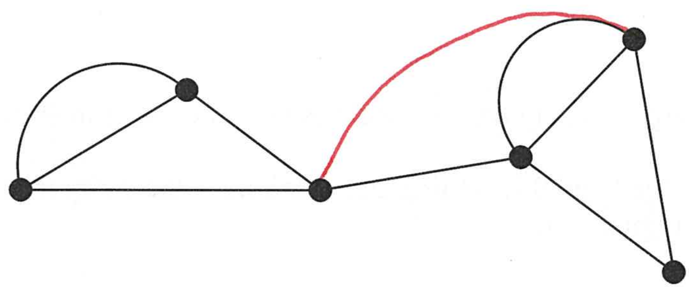
5. (4 marks)

The graph below shows the walkways connecting exhibits at a Zoo.



(a) Define the graph as Hamiltonian, Semi-Hamiltonian or Neither. [1]

Semi - Hamiltonian

(b) How is it possible to tell that the graph does not contain an Eulerian Circuit or a Semi-Eulerian Trail? [1]

It contains more than two vertices with an odd degree.

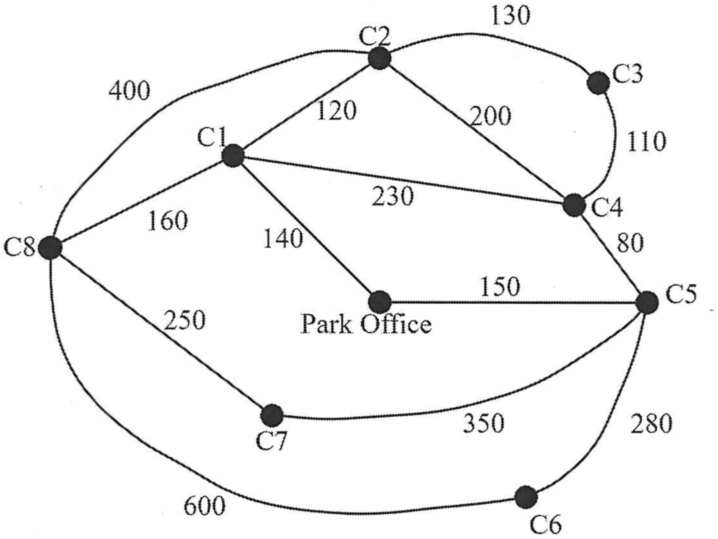
(c) Draw one edge on the graph so that it will now contain a Semi-Eulerian Trail and will no longer contain a bridge. [2]

✓ joins 2 odd vertices

✓ No longer has a bridge

6. (11 marks)

The diagram below shows a network of walking tracks between various campsites and the office in a national park. The lengths of the tracks in metres are also shown.



✓ Path starts at C8 and ends at PO

✓ Visits all stated sites

✓ Shortest Path

(a) A ranger at C8 plans to visit C1, C3 and C7 before returning to the park office. What is the shortest distance he will have to travel? State the route the ranger must take. [3]

1180 ✓ - C8, C7, C5, C4, C3, C2, C1, P.O. ✓ ✓

(b) Each day, the ranger on duty has to inspect each of the tracks to ensure they are safe.

(i) Is it possible to do this starting and finishing at the Park Office without repeating a track? Explain why. [2]

Yes ✓ there are no odd vertices which means it will contain an Eulerian Circuit ✓ OR its possible to traverse every edge once & return to start.

(ii) What is the name for the type of route mentioned in part (b) (i)? [1]

Eulerian Circuit

(c) With the present layout of tracks, it is not possible for a ranger to start at the Park Office, inspect each campsite once without repeating a campsite, and return to the Park Office.

(i) Suggest where an additional track could be added to solve this problem. [1]

C7 to PO

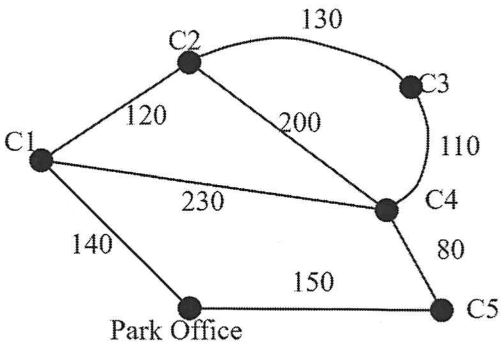
(ii) With this new track, write down a route the ranger could follow to start at the Park Office, inspect each campsite exactly once and return to the Park Office. [2]

PO, C1, C2, C3, C4, C5, C6, C8, C7, PO

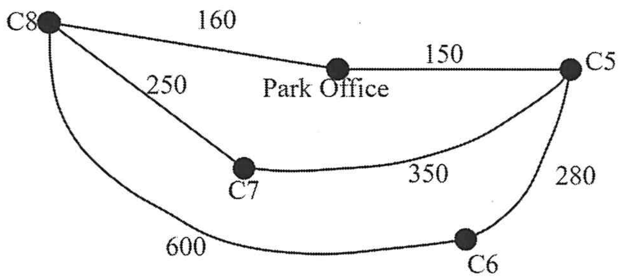
visits all sites once ✓

Returns to PO ✓

(d) Rangers decide to break the map up into 'subgraphs' to show the campsites that a Ranger is responsible for monitoring during their shift. Which of the following is NOT a correctly constructed subgraph? Explain your answer. [2]



Subgraph 1



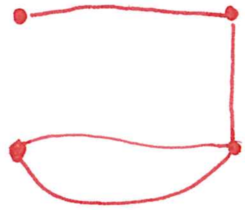
Subgraph 2

Subgraph 2 ✓

Edge from C8 to PO has been added ✓

7. (9 marks)

(a) Draw a connected graph that has four vertices, four edges, two faces and two bridges. [2]



two features correct ✓

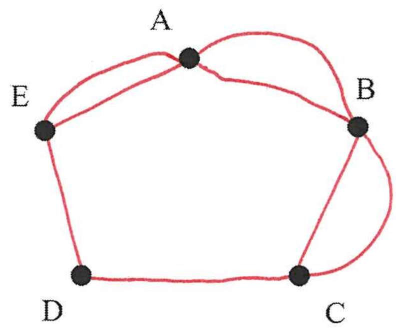
All features correct ✓

(b) Determine the number of edges for a planar graph that has five faces, and five vertices. [2]

$5 + 5 - e = 2$ ✓

$e = 8$ ✓

(c) Draw the graph from part (b) in the space below, clearly showing that it's planar. [2]



Planar ✓

v, f, e correct ✓

(d) State the definition of a Hamiltonian Cycle AND give an example of a Hamiltonian Cycle for your graph in part (c). [2]

A cycle (or path) that includes every vertex exactly once but then returns to the starting vertex.

A, B, C, D, E, A

(e) A simple connected graph contains n vertices. Determine the minimum number of edges it contains. [1]

$n - 1$